



① 10nc @ (0,0,0)

-5nc @ (1,0,0)

-5nc @ (1,1,1)

$$\vec{r} = \vec{a}_y + \vec{a}_z$$

$$10nc \quad \vec{r}' = 0 \quad \vec{r} - \vec{r}' = \vec{a}_y + \vec{a}_z \quad |\vec{r} - \vec{r}'| = \sqrt{2}$$

$$\vec{E} = \frac{10nc}{4\pi\epsilon_0} \left( \frac{\vec{a}_y + \vec{a}_z}{z^{3/2}} \right)$$

$$-5nc \quad \vec{r}' = \vec{a}_x \quad \vec{r} - \vec{r}' = -\vec{a}_x + \vec{a}_y + \vec{a}_z \quad |\vec{r} - \vec{r}'| = \sqrt{3}$$

$$\vec{E} = \frac{-5nc}{4\pi\epsilon_0} \left( \frac{-\vec{a}_x + \vec{a}_y + \vec{a}_z}{z^{3/2}} \right)$$

$$-5nc \quad \vec{r}' = \vec{a}_x + \vec{a}_y + \vec{a}_z \quad \vec{r} - \vec{r}' = -\vec{a}_x \quad |\vec{r} - \vec{r}'| = 1$$

$$\vec{E} = \frac{-5nc}{4\pi\epsilon_0} \left( \frac{-\vec{a}_x}{1} \right)$$

so

$$\vec{E}_{tot} = \frac{5nc}{4\pi\epsilon_0} \left[ \left( \frac{1}{3^{3/2}} + 1 \right) \vec{a}_x + \left( -\frac{2}{2^{3/2}} - \frac{1}{3^{3/2}} \right) \vec{a}_y + \left( \frac{2}{2^{3/2}} - \frac{1}{3^{3/2}} \right) \vec{a}_z \right]$$

$$= 45 \text{ V.m} ( 1.19 \vec{a}_x + 0.515 \vec{a}_y + 0.515 \vec{a}_z )$$

$$= 53.6 \vec{a}_x + 23.1 \vec{a}_y + 23.1 \vec{a}_z \quad \frac{\text{V}}{\text{m}}$$

(1a)

The potential is also found through superposition

10nC

$$V_1 = \frac{10nC}{4\pi\epsilon_0} \frac{1}{\sqrt{2}}$$

-5nC

$$V_2 = \frac{-5nC}{4\pi\epsilon_0} \frac{1}{\sqrt{3}}$$

-5nC

$$V_3 = \frac{-5nC}{4\pi\epsilon_0} \frac{1}{1}$$

So

$$V = \frac{5nC}{4\pi\epsilon_0} \left[ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} - 1 \right]$$

so

$$V = \frac{5nC}{4\pi\epsilon_0} (-0.163)$$

$$= -7.33 \text{ V}$$

(2)

2. a)

$$\vec{E}(r, \theta, \varphi) = E_r(r) \hat{a}_r$$

$$b) \rho = \rho_0 (ar - r^2)$$

$$\epsilon_0 \oint_S \vec{E} \cdot d\vec{s} = Q_{\text{encd.}}$$

use a spherical surface of radius  $r$  for  $S$

$$\Rightarrow \epsilon_0 4\pi r^2 E_r = Q_{\text{encd.}}$$

outside the sphere

$$Q_{\text{encd.}} = \iiint_{0 \leq r \leq a} \rho_0 (ar - r^2) r^2 \sin\theta dr d\theta d\phi$$

$$\begin{aligned} \Rightarrow Q_{\text{encd.}} &= 4\pi \rho_0 \int_0^a ar^3 - r^4 dr \\ &= 4\pi \rho_0 \left( \frac{a}{4} r^4 - \frac{1}{5} r^5 \right) \Big|_0^a \\ &= 4\pi \rho_0 a^5 \left( \frac{1}{4} - \frac{1}{5} \right) = \frac{4\pi \rho_0 a^5}{20} = \frac{\pi \rho_0 a^5}{5} \end{aligned}$$

so

$$E_r = \frac{\rho_0 a^5}{20 r^2 \epsilon_0}$$

(3)

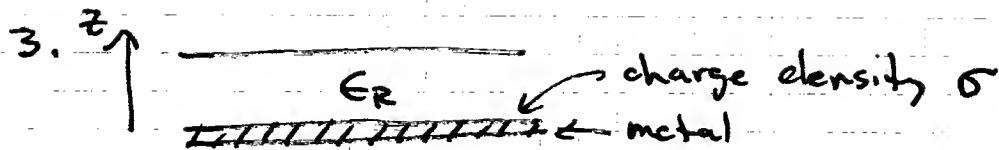
Inside the sphere

$$Q_{\text{encl.}} = \iiint_0^{\pi} \rho_0 (ar - r^2) r^2 \sin\theta dr d\theta d\phi$$

$$= 4\pi \rho_0 \left( \frac{9}{4} r^4 - \frac{1}{5} r^5 \right) \Big|_0^r$$

$$= 4\pi \rho_0 \left( \frac{9}{4} r^4 - \frac{1}{5} r^5 \right)$$

$$\Rightarrow E_r = \frac{4\pi \rho_0 \left( \frac{9}{4} r^4 - \frac{1}{5} r^5 \right)}{4\pi \epsilon_0 r^2} = \frac{\rho_0}{\epsilon_0} \left( \frac{9}{4} r^2 - \frac{1}{5} r^3 \right)$$

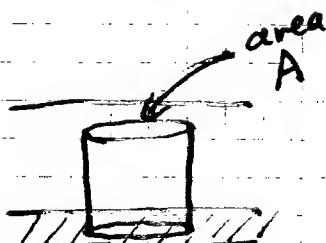


Use Gauss's Law to find  $\vec{D}$  which depends only on the free charge  $\sigma$ .

$$\text{By symmetry } \vec{D} = D_z(z) \hat{a}_z$$

$$\oint_S \vec{D} \cdot d\vec{s} = Q_{\text{encl.}}^{\text{free}}$$

choose for  $S$



only contribution to the surface integral comes from the top plate of  $S$

$$\Rightarrow AD_z = A\sigma$$

$$\Rightarrow D_z = \sigma$$

so

$$\vec{D} = \sigma \hat{a}_z \quad \text{This is the displacement field every where above the metal}$$

Inside the dielectric

$$\vec{E} = \frac{\vec{D}}{\epsilon_0 \epsilon_r} = \frac{\sigma}{\epsilon_0 \epsilon_r} \hat{a}_z$$

Outside the dielectric

$$\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{\sigma}{\epsilon_0} \hat{a}_z$$

The discontinuity in  $\vec{E}$  is produced by a surface layer of bound charge at each surface of the dielectric.

since

$$Q_{\text{total}} = Q_{\text{free}} + Q_{\text{bound}}$$

for the surface S

$$Q_{\text{bound}} = \epsilon_0 \left( \int_S \vec{E} \cdot d\vec{s} - \int_S \vec{D} \cdot d\vec{s} \right)$$

$$Q_{\text{enc}}^{\text{bound}} = \frac{\sigma}{\epsilon_R} A - \sigma A = \sigma A \left( \frac{1}{\epsilon_R} - 1 \right)$$

so the bound charge density on the surface of the dielectric nearest the metal is

$$\sigma_{\text{bound}} = \sigma \left( \frac{1}{\epsilon_R} - 1 \right)$$

a corresponding  $\sigma_{\text{bound}}$  is on the other surface but of opposite sign.

4. a) The half plane will intercept  $1/4$  of the total flux.

Since the total flux of a point charge is just  $Q$  the flux through the  $1/2$  plane is

$$\frac{100 \mu C}{4} = 25 \mu C$$

b)

air cap.

$$\vec{D} = \sigma \vec{a}_z$$

$$\vec{E} = \frac{\sigma}{\epsilon_0} \vec{a}_z$$

$$V = \frac{\sigma}{\epsilon_0} d$$

dielectric cap.

$$\vec{D} = \sigma \vec{a}_z$$

$$\vec{E} = \frac{\sigma}{\epsilon_0 \epsilon_R} \vec{a}_z$$

$$V = \frac{\sigma}{\epsilon_0 \epsilon_R} d$$

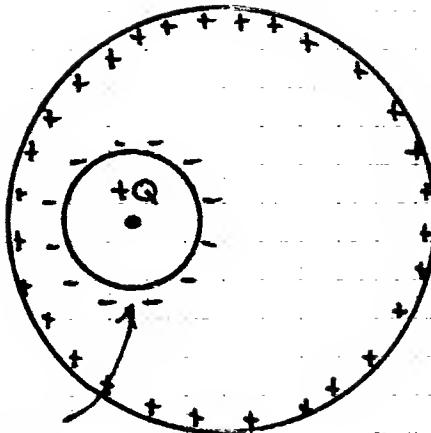
same

air cap greater

air cap greater

(6)

5. a)



uniform + charge  
on metal's surface  
total charge +Q

Uniform -  
charge on inner  
surface of cavity.  
total charge - Q

Inside Cavity:  $\vec{E}$  is just that of the point charge

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r$$

where the origin is at Q

Inside metal  $\vec{E} = 0$

Outside metal

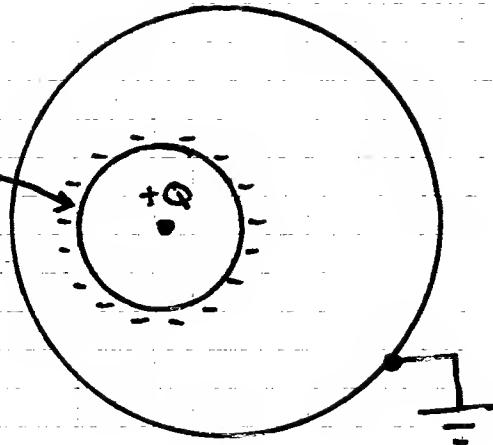
$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r$$

But now the origin is at the center of the metal sphere.

7

b)

uniform -  
charge on inner  
surface of cavity  
total charge  $-Q$



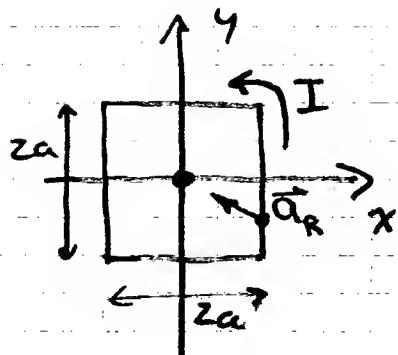
No other charge in metal

Inside cavity  $\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r$  origin at Q

Outside metal  $\vec{E} = 0$

Inside metal  $\vec{E} = 0$

6.



$$\vec{H} = \oint \frac{I d\vec{l} \times \hat{a}_R}{4\pi R^2}$$

Consider the segment at  $x = +a$

the magnetic field from this segment

is

$$\vec{H}_1 = \int I \frac{d\vec{l} \times \hat{a}_R}{4\pi R^2}$$

$$d\vec{l} = \hat{a}_y dy$$

$$\hat{a}_R = \frac{-a \hat{a}_x - y \hat{a}_y}{(a^2 + y^2)^{1/2}}$$

so  $d\vec{l} \times \hat{a}_R = \frac{a \hat{a}_z}{(a^2 + y^2)^{1/2}} dy$

and

$$R = (a^2 + y^2)^{1/2}$$

(8)

so  $\vec{H}_1 = \frac{I}{4\pi} \int_{-a}^a \frac{a dy}{(a^2 + y^2)^{3/2}} \vec{a}_z$

$$= \frac{Ia}{4\pi} \left[ \frac{y}{a^2(a^2 + y^2)^{1/2}} \right]_a^a \vec{a}_z$$

$$= \frac{Ia}{4\pi} \frac{2a \vec{a}_z}{a^2(a^2 + a^2)^{1/2}} = \frac{2I}{\sqrt{2} a 4\pi} \vec{a}_z$$

A moment's thought should convince you that the other three segments produce the same field vector at the center of the loop.

So  $\vec{H} = \frac{2I}{\sqrt{2} a 4\pi} \vec{a}_z$